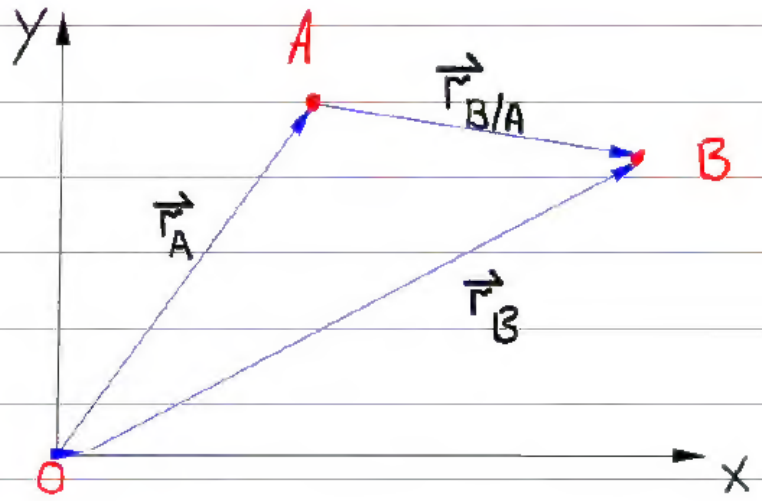
Meriam 2/8 : Relative Motion



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$ ★

where $\vec{r}_{B/A}$ is a vector from A to B
(pretend that you are standing at A, looking at B)

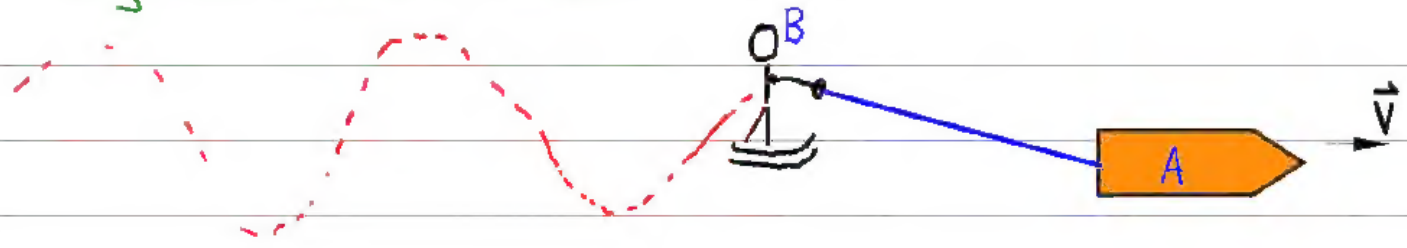
Take time derivatives of ★

→ $$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

These equations can sometimes be used to simplify a complex problem by splitting it into multiple simpler problems.

Eg. A water skier:



Eg: 2 airplanes, A & B

$\vec{V}_A = 700$ km/hr (East)

$|\vec{V}_{B/A}| = 1125$ km/hr

$|\vec{V}_B| = 500$ km/hr

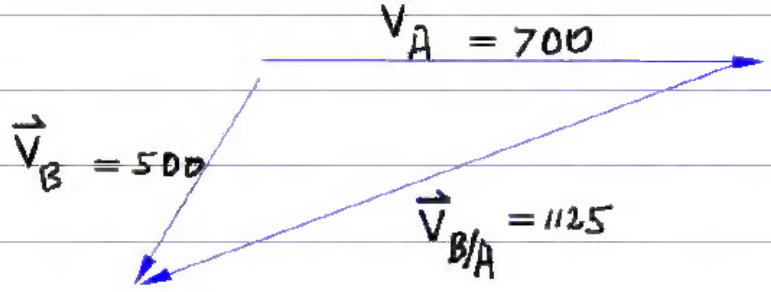
The direction of B is known to be somewhere in the southwest quadrant

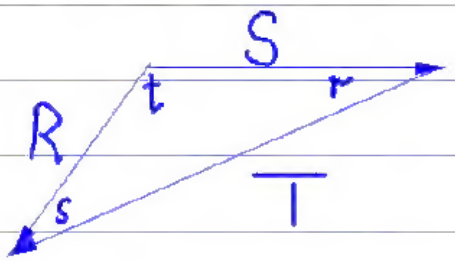
Find the exact direction of B

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

This is a 2D vector equation, equivalent to two scalar equations with 6 variables

4 variables are known → solve for remaining two





$$T^2 = R^2 + S^2 - 2RS\cos(t)$$

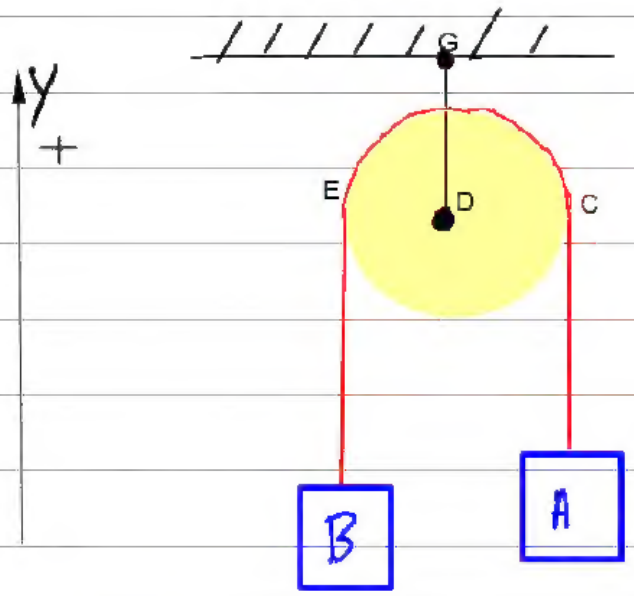
$$1125^2 = 500^2 + 700^2 - (2)(500)(700)\cos(t)$$

$$\cos(t) = -0.7509$$

$$t = 139^\circ \quad (\text{or } 221^\circ)$$

Pulleys:

Assume that ropes do not stretch or go slack or bunch up.



Find $\vec{V}_C, \vec{V}_E, \vec{V}_B$ if $\vec{V}_A = +0.5\hat{j}$ m/s

Note: Yellow disk is a solid body, where

$\vec{V}_G = \vec{V}_D = 0$ $\quad \vec{V}_A = \vec{V}_C$ & $\vec{V} = \vec{V}_E$

Position of D is halfway between E & C

ie, Position of D is average of positions of E & C

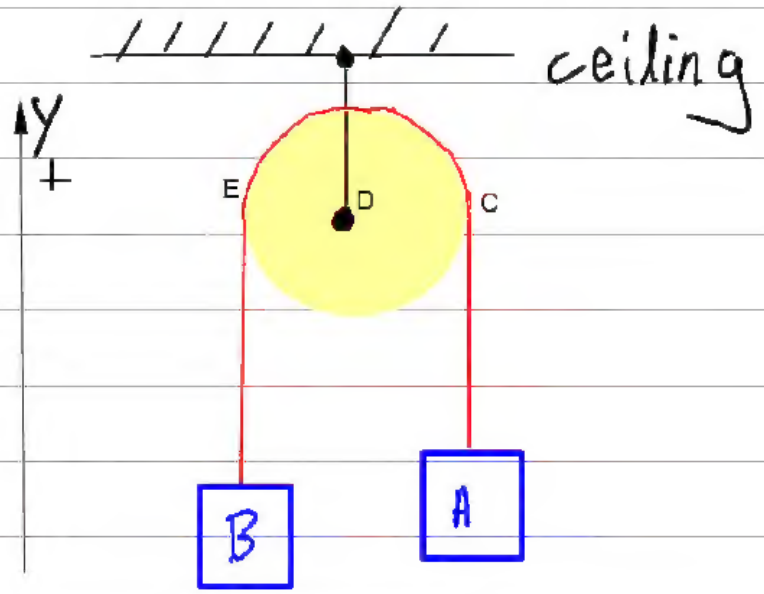
$$\vec{y}_D = \frac{\vec{y}_C + \vec{y}_E}{2} \; ; \quad \vec{V}_D = \frac{\vec{V}_C + \vec{V}_E}{2} \; ; \quad \vec{a}_D = \frac{\vec{a}_C + \vec{a}_E}{2}$$

$$0 = \frac{0.5\hat{j} + \vec{V}_E}{2} \rightarrow \vec{V}_E = \vec{V}_B = -0.5\hat{j}\ \text{m/s}$$

Repeat the problem from page 7(c), but now there is an earthquake.

Find $\vec{V}_C, \vec{V}_E, \vec{V}_B$ if $\vec{V}_A = 0.5\hat{j}$ m/s & $\vec{V}_G = -0.7\hat{j}$ m/s

Answer:



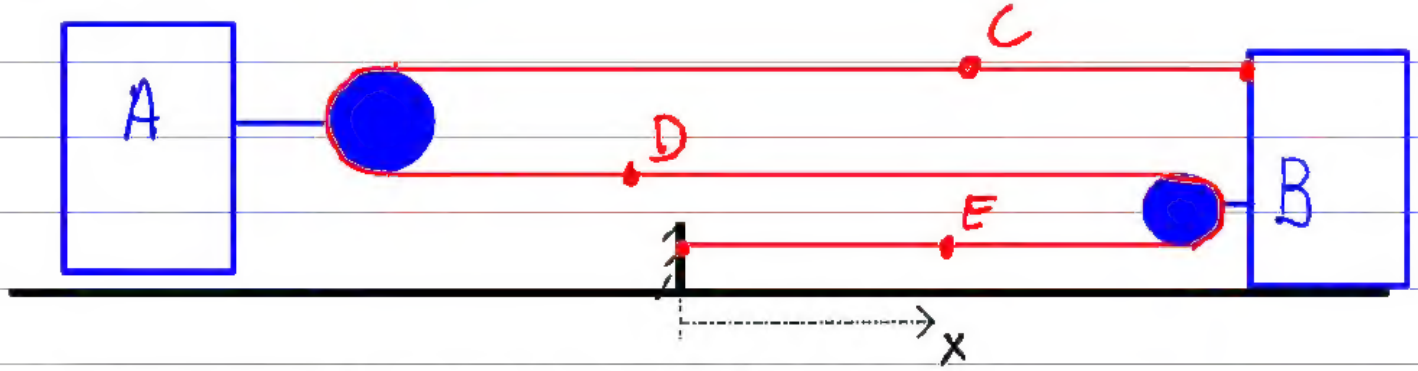
$$\vec{V}_D = \frac{\vec{V}_C + \vec{V}_E}{2}$$

$\vec{V}_G$, $\vec{V}_A$, $\vec{V}_B$

$$-0.7\hat{j} = \frac{0.5\hat{j} + \vec{V}_B}{2} \rightarrow \boxed{\vec{V}_B = -1.9\hat{j}\ \text{m/s}}$$

A more challenging example...

Eg.



Given: $\vec{V}_B = +0.45\,\hat{i}$ m/s

Find: $\vec{V}_A$, $\vec{V}_D$, $\vec{V}_{A/B}$, $\vec{V}_{C/D}$, $\vec{V}_E$, $\vec{V}_C$

Answer: $\vec{V}_E = 0$; $\vec{V}_C = \vec{V}_B = 0.45\,\hat{i}$ m/s

Right wheel:

$$\vec{V}_B = \frac{\vec{V}_D + \vec{V}_E}{2} \quad (0.45\,\hat{i},\ 0) \rightarrow \vec{V}_D = 0.9\,\hat{i} \text{ m/s}$$

Left wheel:

$$\vec{V}_A = \frac{\vec{V}_C + \vec{V}_D}{2} \quad (0.45\,\hat{i},\ 0.9\,\hat{i}) \rightarrow \vec{V}_A = 0.675\,\hat{i} \text{ m/s}$$

$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B = 0.675\hat{i} - 0.45\hat{i} = 0.225\hat{i} \text{ m/s}$$

$$\vec{V}_{C/D} = \vec{V}_C - \vec{V}_D = 0.45\hat{i} - 0.90\hat{i} = -0.45\,\hat{i} \text{ m/s}$$

ie. C is falling behind D

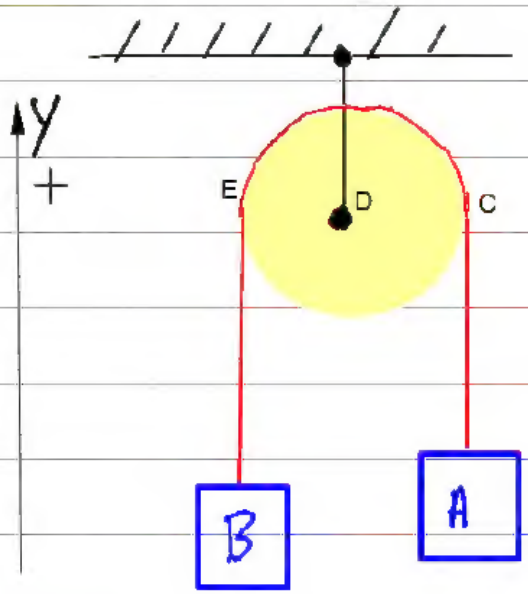
Summary

Relative Motion:

- $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$
- $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$
- $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

Wheels & pulleys:

- $\vec{y}_D = \dfrac{\vec{y}_C + \vec{y}_E}{2}$
- $\vec{v}_D = \dfrac{\vec{v}_C + \vec{v}_E}{2}$
- $\vec{a}_D = \dfrac{\vec{a}_C + \vec{a}_E}{2}$



For Next Lecture:

- Read 2/9, 3/1 – 3/4